An approach to building thinking classrooms in Year 7 mathematics


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With the aim of engaging more students in meaningful thinking in mathematics, a cohort of Year 7 students were introduced to a 'Building Thinking Classrooms' framework for learning in mathematics. In this paper, the authors outline the key features of the implemented framework and share reflections on challenges and benefits.

The importance of focusing on ways to improve or enhance student thinking in mathematics is well established in the literature (e.g. Ball & Cohen, 1999). Resnick (1989, cited in Schoenfeld 2016, p. 7) suggests that becoming a good thinker in mathematics "may be as much a matter of acquiring the habits and dispositions of interpretation and sense-making as of acquiring any particular set of skills, strategies, or knowledge". This suggests that creating the right conditions, that is, the right environment, for mathematical thinking is just as important as the actual mathematical content itself. Liljedahl's (2020) *Building Thinking Classrooms* framework is built on this premise — that the conditions for particular types of practices are essential.

In his early observations of mathematics lessons, Liljedahl (2016) estimated that about 20% of students were thinking for about 20% of a lesson. To counter this, Liljedahl proposed 14 practices, which collectively form the Building Thinking Classrooms framework, to disrupt institutional norms and expectations of what mathematics lessons 'should look like' (Liljedahl, 2020). The intention of Liljedahl's framework is to refocus mathematics lessons to create more time, opportunities, and expectations for student thinking. The list includes classroom practices such as the use of vertical non-permanent surfaces, visibly random groupings, rich mathematical tasks, the timing and delivery method of tasks, and how students' questions are answered.

Hiebert and Grouws (2007) highlight how, despite decades of research on improving the mathematical experience of students, there are still many challenges — including the everyday pressures and realities teachers face. These pressures can come from school or system leadership, parents, and students, many of whom have fixed views and expectations of what mathematics classrooms should look like. Hence, the lead author did not opt for a radical overhaul of her classroom, as was the case described by Liljedahl (2016). Instead, she chose to implement a series of smaller changes in the hopes of them becoming sustained practices.

In this paper we provide an overview of the approach taken by the lead author to implementing the Building Thinking Classrooms framework with a Year 7 cohort and summarise the key challenges and benefits. The approach was embedded across a series of lessons over the course of a term, which was later expanded throughout the remainder of the year. In this paper, we draw on examples from one specific lesson. The approach was centred on the use of four pedagogical moves: vertical non-permanent surfaces, visibly random groupings, rich mathematical tasks, and prompts.

Vertical non-permanent surfaces

Vertical non-permanent surfaces (VNPS) are vertical surfaces, such as whiteboards, blackboards, or windows on which students do their work, using erasable materials like non-permanent markers or chalk. According to Liljedahl (2020), VNPS offer benefits for both teachers and students. For teachers, VNPS provide greater visibility of student thinking, allowing them to more easily scan and check on students' progress. This increased visibility enables teachers to make more efficient decisions about when to intervene and support students, and which students might require extra assistance.

For students, the non-permanent nature of white-boards or wipeable surfaces creates a lower-stakes environment. Mistakes can be easily erased on VNPS, which, as Liljedahl (2020) emphasises, encourages students to take more risks when working on challenging tasks or problems. In other words, students are more likely to start working and 'trying things out' with wipeable vertical surfaces compared to working in books or on chart paper. Students also appear to put less pressure on themselves to write perfectly or present fully polished ideas and processes.

Liljedahl (2020) also highlights that students often feel anonymous when sitting and working at desks. In contrast, when standing at VNPS, they adopt a different physical presence and role in the classroom, which leads to greater involvement in learning. This heightened engagement lends itself to greater collaboration among students. As they stand around VNPS, students can easily see each other's work, offer suggestions, ask questions, and engage in meaningful discussions about the tasks they are working on.

Visibly random groupings

Many educators advocate for collaborative or group work approaches in the classroom. Liljedahl (2016) investigated different grouping strategies and found a profound change in classroom dynamics when groupings are not only random but also made 'visibly' in front of students so they can see that the groupings are genuinely random. Liljedahl (2020) observed that when visibly random groupings were used daily with students in middle school mathematics classrooms students became agreeable to work in any group they were placed in, there was an elimination of social barriers within the classroom and, as a result, mobility of knowledge between students increased. In turn this meant reliance on the teacher for answers decreased, engagement in classroom tasks increased, and students became more enthusiastic about mathematics class.

Rich mathematical tasks

The use of 'rich' tasks is often encouraged as a way to foster student engagement in mathematical thinking and activity. While the term is commonly used in mathematics education, there is no consensus of what constitutes a 'rich' task. However, there are common characteristics across different definitions. These include invoking student knowledge in ways that have not been routinised (Liljedahl, 2020), being mathematically accessible to all students and with built-in extension opportunities (the 'low-threshold high-ceiling' concept promulgated by NRICH), engaging students in inquiry-based learning (Van de Walle, 2019), and providing students with challenge (Sullivan, 2015). Other features mentioned by some definitions include promoting discussion and offering multiple solution pathways, which in turn lend themselves to greater opportunities for within-task differentiation (Piggott, 2018) .

Prompts

Two of Liljedahl's practices (2016), Practice 5 'How we answer questions' and Practice 9 'How we use hints and extensions to further understanding', overlap with Sullivan et al.'s work (2006) on the use of prompts to promote more challenge and thinking in mathematics classrooms. Sullivan et al. (2006) encourage the use of both enabling and extending prompts. Both types of prompts can be used to differentiate within the task. Enabling prompts increase accessibility with modifications or questions that are still carefully connected to the main task. An enabling prompt may become the new focus for students or a catalyst for their thinking so they can return to the original task. Extending prompts are intended to provide additional challenge with a higher cognitive demand. Like enabling prompts, extending prompts remain connected to the original task and draw on similar reasoning, conceptualisations, and representations (Sullivan et al., 2006).

The study

The lead author, as part of a year-long professional learning program aimed at supporting middle school mathematics teachers to embed more rich mathematical experiences into everyday teaching and learning, introduced a modified Building Thinking Classrooms approach with her Year 7 mathematics class. The aim was to provide students with a safe environment to take risks in their mathematical thinking and to foster a growth mindset in their mathematical problem solving. Specifically, and as discussed in this paper, she incorporated (1) visibly randomised groups of three students, (2) vertical non-permanent surfaces in the form of vertical whiteboards, (3) the use of rich mathematical tasks, and (4) enabling and extending prompts.

A Year 7 class participated in the study, which took place at a secondary college (Years 7–12) in a low socio-economic outer suburban area of Adelaide. The Year 7 class comprised 27 students and had a diverse range of learners including several students who have English as an Additional Language or Dialect (EALD). Previous and on-going assessment data also pointed to a range of mathematical levels across the class (from Year 3 to Year 10). The students had no known prior experience working on rich mathematical tasks in randomised groups.

While a range of different rich mathematical experiences and activities were used across Term 3, informed by the year of professional learning, this paper focuses on Building Thinking Classrooms strategies embedded across a unit of work with Year 7 students. One particular lesson has been selected as an example to detail the lead author's approach. This lesson was chosen by the teacher, thinking it might appeal to the students by referencing popular culture. However, as described later, this was not the case.

The rich mathematical task on which this lesson was based is a task called *How Much Bigger Should They Make Zoolander's School?* from Robert Kaplinsky (2014, np). Kaplinsky describes the problem as being:

> …about a male model named Derek Zoolander (played by Ben Stiller) who means well but is not the sharpest tool in the shed. In this scene he learns about a school that will be built in his name. He doesn't realize that he is being shown a model of the school and not the actual school itself and angrily tosses it to the ground. He then states, after much thinking, that it needs to be three times bigger. While his answer is technically not wrong (he said "at least"), it is a relatively inaccurate guess.

In this problem, students need to determine how many times bigger the model of the school should be. All four key pedagogical strategies from the Building Thinking Classrooms approach framed the structure of this lesson.

Overview of the lesson

The lesson was launched by showing a short video (provided in Kaplinsky, 2014) to set the scene. Students were asked to share what they noticed and what they wondered (a routine that was by now well-established in this class), which created a safe space for students to begin engaging with the task and begin their thinking process about mathematical questions that could be asked about the situation.

While students were initially reluctant to suggest possible mathematical questions relating to the scenario, after some prompting by the teacher, the question "How much bigger should they make the school?" was quickly established. Before moving on to begin solving the problem, students were placed in visibly random groups. For this particular task, cards were used to group students. Each student picked a card at random and moved to a vertical whiteboard that displayed the background colour of their card. Each student then attached their grouping card to their group's whiteboard. The cards were later used as tokens that could be exchanged for prompt cards, meaning that each group was able to access up to three enabling prompts if they wished. A bank of prompts specific to the task was pre-prepared (see Figure 1). The lead author had previously used this strategy of exchanging 'tokens' for prompts with this class and had found that 'exchanging' tended to give more value to the prompts with students reluctant to 'cash-in' their tokens too early.

Students worked on the problem while the teacher moved around the room to observe student thinking and prompt discussion. The lesson was

What is a number that is too low? Too high?	How big would the school be if it were twice as big as the model?	How big would the school be if it were five times as big as the model?
How big do you think the model is?	How big do you think a real school of this design would be?	How many stories high is the school in the model?
What body parts of Derek Zoolander could you use to determine the dimensions of the model?	Since we don't have the information we need, what other methods could we use to find out how much bigger the school should be?	How could you use the figures of humans in the model to work this out?

Figure 1. Pre-prepared enabling prompts for the Zoolander task.

recorded using a GoPro 360° camera so that the teacher could go back and reflect on the lesson at a later point as part of the professional learning program. Students worked through the task mostly as a group with varying degrees of success amongst groups. At the end of the lesson, students were given a template for self-reflection on the lesson.

Key findings: challenges with groupings and with prompts

When students moved to their whiteboards with their groups, they were slow to begin working on the problem. Most students were unable to connect the problem established in the whole class discussion (*How much bigger should they make Zoolander's school?*) with what they were expected to do on their whiteboards. In fact, many groups began attempting solve the 'problems' they thought they noticed in the top left-hand corner of the cards that were used for grouping in this lesson. This appeared to be because they were more used to visibly random groups being created using online random grouping software, and so were unsure about the purpose of specific details on the grouping cards. Based on this experience, using online grouping software seems to be more efficient timewise, eliminating the need for managing physical grouping cards and removing the distraction of the cards.

Once the distraction of the cards was overcome, it was evident that most groups were unable to come up with an appropriate strategy on which to base their mathematical reasoning. Student confidence appeared to be a key factor in students' success at attempting the task and showing any kind of thinking on their whiteboard. The enabling prompts were intended to nudge students' thinking by giving them something to think about when they weren't sure where to start. Upon reflection, the lead author thinks the prompts appeared to provide only limited help to students in effectively working through the problem. Most students seemed unable to make meaningful connections between the prompt and the problem. If students were able to make meaning from the prompt, they struggled to link the prompt with where their thinking could go next. The most effective types of prompts were more closed questions such as "how big would the school be if it were … times as big as the model?", which meant students could use a trial-and-error method to produce a solution.

One group did appear to successfully use the prompts. Figure 4 compares thinking before and after using prompts. The image on the left shows a diagram of the mathematical scenario and some repeated addition showing the students' attempt to do 'something' with the numbers given in the problem. In contrast, the image on the right shows evidence of further labelling information on the diagram and the use of multiplicative thinking, suggesting that the prompts helped students to keep working on the problem.

Although having pre-prepared prompts was a helpful resource in the busy classroom environment, we believe there may be more effective strategies for encouraging students' thinking without limiting their ideas or confusing them with ones they perceive as irrelevant to the problem. To effectively foster engagement in the thinking process, it may be better for students to come up with their own ideas. If students are truly stuck in their problem solving, then it could be that the problem is too far beyond their zone of proximal development. In this case the problem itself needs to be changed, or an alternative one needs to be presented.

Key findings: providing students with challenge

More generally, it was interesting that students who tended to struggle in a more traditional style classroom, often due to factors such as learning

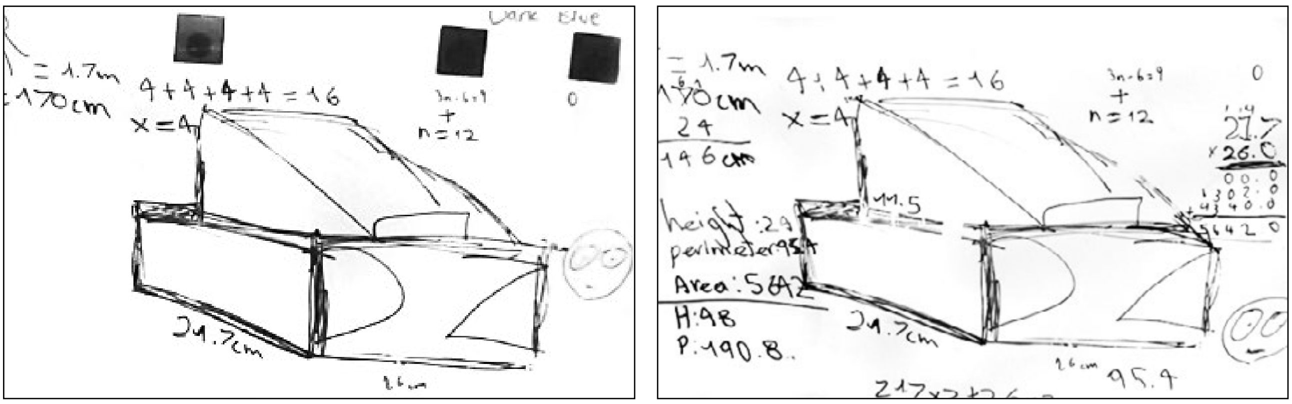


Figure 2. Example of one group's thinking before using any prompts (on the left) versus after using prompts (on the right).

difficulties or low confidence in mathematics, were highly engaged in rich tasks that utilised VNPS and visibly random groupings. Conversely, students who were generally comfortable with the traditional classroom model found the Building Thinking Classrooms approach particularly challenging. We believe the Building Thinking Classrooms approach is important for students who may not be sufficiently extended in the traditional classroom and are not pushed to extend their thinking beyond tasks that can be solved with routine methods. These students are often focussed on 'finding the answer' rather than exploring different problem-solving techniques or considering alternative solutions, outcomes, or contexts to problems.

The majority of the rich mathematical tasks used connected to very specific mathematics curriculum learning outcomes (e.g., estimating, rounding, or expressing ratios and fractions). However, some of the tasks used could be described as non-curricular. In these non-curricular rich tasks, the learning intentions were varied, ranging from showing collaborative skills, using different problem-solving techniques, reflecting on feelings around completing a mathematics activity, or reinforcing previously learned mathematical skills. The use of rich tasks in a non-curricular way provided an experience of mathematics that was different from what students may have previously engaged in, in terms of the type of task, its presentation, and the method of making thinking visible. Hence, the formative assessment used at the end of a lesson could be captured through methods such as student self-reflection (completed for this lesson a self-reflection form, as shown in Figure 3) or a brief oral presentation of thinking.

A key advantage of the self-reflection was that it enabled students to reflect on their own dispositions. For instance, students' reflections sometimes focused on their feelings during the task and their perceptions of how well they felt they were working with their group members. Some examples are shown below.

"A new skill I learnt today" is:

- "Working with different people and seeing the way they saw things instead of me."
- "Improving speaking in front of the class."
- "Not getting rid of my work because it's a habit for me to erase my work."
- "To work with other/different people. I learnt how they think, and all their ideas. I learnt how to do stuff like they do."

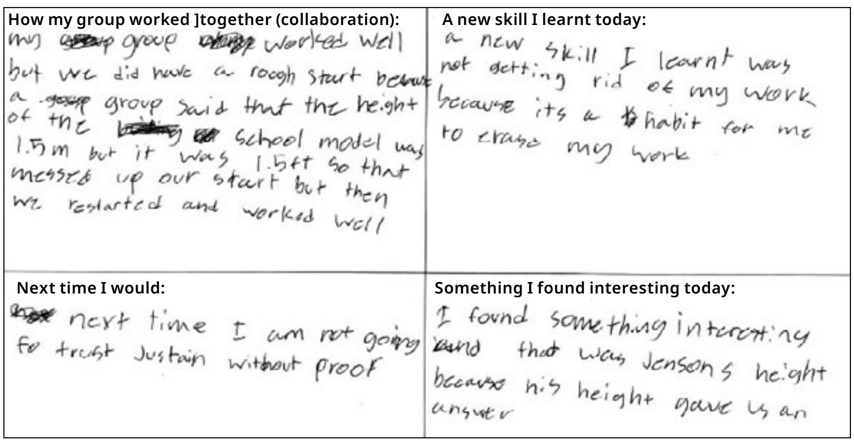
"Something I found interesting today" is:

- "How many people got the same answer."
- "People in my class who I don't really work well with."
- "I found working with new people interesting because we all have different skills, and I actually found out that my classmate is pretty smart. I also found the video interesting."
- "The use of cards for clues."
- "Jenson's height because his height gave us the answer."

Key findings: planning and teaching

Rich mathematical tasks

The importance of a task's relevance and connection to the students became apparent with the use of the Zoolander Task. Students did not find the task particularly engaging as they were unfamiliar with the movie and didn't quite understand the relevance of determining how much bigger the school would have to be. This experience suggests that a task with a less specific context might have less distraction from the underlying mathematical



How my group worked]together (collaboration):
my group worked well but we did have a rough start becau a group said that the height of the school model was 1.5m but it was 1.5ft so that messed up our start but then we restarted and worked well

A new skill I learnt today:
a new skill I learnt was not getting rid of my work because its a habit for me to erase my work

Next time I would:
next time I am not going to trust Justain without proof

Something I found interesting today:
I found something interesting and that was Jensons height because his height gave us an answer

Figure 3. Example of a self-reflection using the provided template.

concepts while still allowing students to personally connect with the activity.

Visibly random groupings

Students seemed to accept and enjoy the opportunity to work with others they were not normally grouped with. Some expressed surprise at how well they worked with someone they were not expecting to connect with or be able to learn from. Additionally, some students who normally relied on the assistance of their close peers to complete tasks struggled with the task as they no longer had this support. This meant many of them had to think more independently and have a go at problems that were challenging for them. On the other hand, students who were typically either quiet in class or had difficulty interpreting written questions seemed to thrive when given the opportunity to work with different students or lead a group of less confident students.

Vertical non-permanent surfaces (VNPS)

Students were initially hesitant when using VNPS, most likely due to the unfamiliarity of working mathematically in such open and visible ways. With encouragement from the lead author, they started by drawing diagrams and labelling what they already knew. It took considerable prompting for students to use the VNPS to record their mathematical working. As a result, a developmental approach was taken to encourage students to build up their confidence in using VNPS. For example, students could choose how to verbally present their groups' problem-solving approaches at the end of a lesson, including the option to share their thinking themselves if they felt confident, or have the teacher do it if the group was still building their confidence. This helped students feel less pressure to share and, given the focus was on approaches rather than solutions, reinforced the value and validity of using different problem-solving approaches, even if the methods did not always lead to a correct solution. The more exposure the class had to VNPS, the more comfortable they became with this way of working.

Prepared bank of prompts

Preparing the prompts in advance was a useful part of the lead author's planning as it allowed them to anticipate possible challenges students might have and identify key opportunities for extension. However, using only the bank of pre-prepared prompts meant students weren't necessarily getting the prompt they really needed when they got 'stuck'. On reflection, it may be more useful to have a variety of pre-prepared prompts that include both closed and open questions, and issue them to students based on their specific needs, as determined at the time by the teacher. This highlights the importance of teachers engaging in a variety of tasks to get to know their students, the way they learn, and what is needed to keep them thinking.

Additional observations

Student metacognition through self-reflection was useful evidence of what students were taking away from the lesson and experience of the Building Thinking Classrooms approach. The nature of the reflection was largely around the way students were working with each other, which was positive evidence of one of the learning intentions of building collaboration skills. However, the student feedback lacked specific evidence of development of mathematical concepts. In addition to reflecting on their experience of the lesson, students also needed explicit reflection prompts to promote the metacognitive aspects of mathematical thinking, especially if the learning intentions focussed on a particular mathematical skill or concept.

The sequence of the lesson was important in allowing students to make connections between one part of the lesson and the next. A useful strategy may have been to have students in their randomised groups at their whiteboards prior to developing the problem-solving question as a class. As these students were early middle-school students without much experience in engaging in rich mathematical tasks in this way, they may have needed greater scaffolding in the earlier sessions to help build the thinking skills required to make connections between the different parts of the problem-solving process.

Conclusion

This paper provides an overview of one Year 7 teacher's experiences with a modified approach to Liljedahl's (2020) *Building Thinking Classrooms*. The approach, based on the work of Liljedahl (2016, 2020), incorporated four key pedagogical moves: vertical non-permanent surfaces, visibly random groupings, rich mathematical tasks, and enabling and extending prompts. The aim was to create a safe environment to encourage students who were disengaged from or lacked enthusiasm for mathematics to take risks in their mathematical thinking and to foster a growth mindset in problem solving.

The study highlighted both challenges and benefits of implementing this approach. Visibly random groupings proved effective in encouraging students to work with different peers and break down social barriers, leading to increased knowledge mobility and reduced reliance on the teacher for answers. However, the physical grouping cards used in the lesson caused some distraction, suggesting that digital grouping tools may be more efficient. While the use of rich mathematical tasks, such as the Zoolander task in this example, provides opportunities for student engagement and thinking, it took some time to build student confidence in engaging with challenging tasks while working at VNPS. The use of VNPS initially posed a challenge for students who were hesitant to work mathematically in open and visible ways. However, with encouragement and support from the teacher over time, students have gradually built their confidence in using VNPS. In this example, enabling and extending prompts were found to have limited effectiveness in nudging students' thinking when they were stuck. While pre-prepared prompts were helpful for the teacher, they did not necessarily target students' specific needs. A more flexible approach, with a variety of open and closed prompts issued based on student needs, may be more beneficial.

In summary, the combination of strategies selected from the Building Thinking Classrooms approach proved useful in shifting many students from passive to active learners, with students coming to mathematics class knowing they would be responsible for communicating their ways of thinking and working.

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